ing as a thinking substance in the process of organic life, the fluid, elastic, plastic ether will reconcile at last materialism and spiritualism on the neutral ground of substantialism. With such ever living, acting and reacting substance, we return to the marvelous conception of the unity of force, substance and mind which the Ionian dynamists had already established more than two thousand years ago!

ARISTIDES PRATELLE.

PARIS, FRANCE.

MAGIC SQUARES MADE WITH PRIME NUMBERS TO HAVE THE LOWEST POSSIBLE SUMMATIONS.

In making magic squares of all orders with prime numbers it is evident that the sum of the series used must be evenly divisible by n; also, that the quotient must be even when n is even and odd when n is odd.

The number 2 is not used in the construction of prime magics, for, being an even number, it has no analogue among the other primes, which are all odd numbers. In seeking for a series of prime numbers suitable for making magic squares *with the lowest possible summations,* if the first n^2 primes will fill the requirements above stated, they will naturally constitute such a series. If, however, the division leaves a remainder $(=r)$, then one or more substitutions must be made among the higher prime numbers so as to increase the total by an amount equal to $n-r$ or $2n-r$ etc., always taking care to secure the smallest increase possible.

Series of prime numbers theoretically suitable for all squares up to and including that of the 12th order showing the lowest possible summations, are given in the following pages, and it is interesting to note that with the exception of squares of the 3d and 4th orders, these predetermined series of prime numbers have all been arranged in magic formation by various experts.

SQUARE OF THE THIRD ORDER.

The sum of the first nine prime numbers 1 to 23 inclusive is 99, which divided by 3 gives 33 as the quotient.

This sum is therefore theoretically suitable, but it can be demonstrated that 111 is the lowest possible summation for a

square of this order made with primes, and the required series is as follows:

1 — 7 — 13 — 31 — 37 — 43 — 61 — 67 — 73

67	1	43
13	37	61
31	73	7

Fig. 1.

By arranging these numbers in accordance with the single rule that is applicable to this order, the square shown in Fig. 1 is produced, which was first made by Mr. Henry E. Dudeney.

SQUARE OF THE FOURTH ORDER.

The sum of the first sixteen prime numbers 1 to 53 inclusive is 380, which is evenly divisible by 4, but the quotient (95) being an odd number, makes the series inadmissible. The lowest sum-

3	71	5	23
53	11	37	1
17	13	41	31
29	7	19	47

Fig. 2.

mation for this square that is known to the writers is 102 (See Fig. 2), the series for which is made by substituting 71 for 43, thus

NUMBERS OF ORIGINAL SERIES OMITTED	NEW NUMBERS ADDED	TOTALS OF NEW SERIES	SUMMATIONS
47	59	392	98
53	73	400	100
41	61	400	100
47, 53	59, 61	400	100
47	67	400	100
43	71	408	102

raising the total of the series to 408. This square was made independently by Mr. Ernest Bergholt and Mr. C. D. Shuldham, Wyoming, New Jersey. There are at least five other series which have

lower totals than 408 and which are theoretically suitable, but it is stated by competent mathematicians that 102 is the lowest possible summation of a square of this order made of prime numbers. These five series are, however, given below together with the one used in the square wherein S = 102, the departure from original series 1 to 53 inclusive being shown.

SQUARE OF THE FIFTH ORDER.

The sum of the first twenty-five prime numbers, 1 to 97 inclusive, is 1059 which is inadmissible. The twenty-fifth prime (97) must therefore be changed to 103, raising the total of the series to

13	61	103	31	5
71	1	17	83	41
23	79	37	7	67
47	29	53	73	11
59	43	3	19	89

Fig. 3.

1065, the fifth part of which is 213. The difficult problem of arranging this series in magic formation was solved by Mr. H. A. Sayles, Schenectady, N. Y., as shown in Fig. 3.

SQUARES OF THE SIXTH ORDER.

The sum of the first thirty-six primes, 1 to 151 inclusive, is 2426 which is inadmissible. By changing the thirty-sixth prime (151) to 173 the total of the series is raised to 2448, which gives

71	19	61	137	79	41
97	59	149	13	37	53
31	131	3	113	23	107
89	67	5	101	139	7
11	103	173	1	47	73
109	29	17	43	83	127

Fig. 4.

127	3	47	23	59	149
83	43	137	73	5	67
19	89	29	61	173	37
97	31	79	107	53	41
71	103	7	13	101	113
11	139	109	131	17	1

Fig. 5.

the quotient 408 when divided by 6. The arrangement of this series of primes in magic order was accomplished independently by Mr.

C. D. Shuldham and Mr. J. N. Muncey, Jesup, Iowa, and their different squares are given respectively in Figs. 4 and 5.

SQUARES OF THE SEVENTH ORDER

The sum of the first forty-nine primes 1 to 227 inclusive is 4887 which is not evenly divisible by 7. By substituting 233 for

139	211	43	83	149	13	61
197	1	19	199	79	157	47
173	181	67	41	71	163	3
37	7	151	89	127	179	109
113	31	131	223	137	11	53
17	167	97	5	107	73	233
23	101	191	59	29	103	193

Fig. 6.

223	3	197	61	11	13	191
79	43	41	233	163	31	109
71	193	53	113	59	137	73
19	179	7	97	149	67	181
139	47	173	167	89	83	1
151	131	101	23	29	157	107
17	103	127	5	199	211	37

Fig. 7.

227 the total of the series is raised to 4893, the seventh part of which is 699.

Messrs. C. D. Shuldham and J. N. Muncey succeeded independently in making different magic arrangements of this series as given respectively in Figs. 6 and 7.

SQUARES OF THE EIGHTH ORDER.

The sum of the first sixty-four prime numbers, 1 to 311 inclusive, is 8892 which is not evenly divisible by eight, but by

271	3	7	11	181	251	83	307
263	43	283	79	31	163	29	223
173	131	67	109	101	37	269	227
179	61	229	197	73	71	167	137
23	211	149	157	151	107	257	59
97	233	199	239	5	191	103	47
89	293	127	41	241	17	193	113
19	139	53	281	331	277	13	1

Fig. 8.

changing 311 to 331, the total of the series is raised to 8912, the eighth part of which is 1114. The magic square shown in Fig. 8 was made from the above series by Mr. J. N. Muncey. Mr. C. D. Shuldham has likewise made a square of this order by using a series

179	149	233	5	157	103	47	241
73	137	227	191	61	23	173	229
181	19	7	223	269	199	89	127
37	151	257	97	31	277	163	101
113	197	43	79	263	29	283	107
271	109	13	211	3	281	167	59
193	311	83	1	17	131	139	239
67	41	251	307	313	71	53	11

Fig. 9.

in which 313 is substituted for 293—which also has a total of 8912. These two squares both showing the lowest possible summation of 1114, are given respectively in Figs. 8 and 9.

SQUARES OF THE NINTH ORDER.

The first eighty-one primes, 1 to 409 inclusive, have a total of 15115. To raise this total to the lowest number evenly divisible by 9,

409	389	73	17	383	11	19	13	367
311	67	5	137	43	331	397	31	359
103	241	53	347	139	227	37	283	251
307	149	233	167	337	127	101	109	151
29	263	349	47	197	401	113	211	71
193	61	239	269	173	179	317	191	59
199	157	277	89	271	41	293	97	257
107	353	79	229	131	83	223	313	163
23	1	373	379	7	281	181	433	3

Fig. 10.

the last number of the series (419) must be changed to 433, thus making the total 15129, the ninth part of which is 1681. Magic squares showing this lowest possible summation were made by Messrs. J.

281	181	37	227	271	103	307	233	41
13	293	383	19	241	7	73	263	389
101	137	3	379	47	337	53	373	251
397	5	71	67	353	277	331	149	31
61	409	163	229	127	179	433	1	79
367	59	191	269	347	131	17	193	107
97	23	311	151	157	89	317	313	223
197	173	283	257	109	199	139	113	211
167	401	239	83	29	359	11	43	349

Fig. 11.

N. Muncey and C. D. Shuldham and are given respectively in Figs. 10 and 11.

SQUARE OF THE TENTH ORDER.

The first one hundred primes, 1 to 541 inclusive, sum 24132. To make a total that is evenly divisible by 10 the last number of the

1	3	431	503	17	509	13	443	467	29
409	67	307	89	47	457	43	499	37	461
79	379	83	397	97	367	211	107	347	349
181	569	163	157	167	317	139	137	227	359
277	179	271	193	199	197	401	331	241	127
283	281	191	269	263	257	233	251	149	239
173	293	53	311	313	131	337	223	353	229
419	151	389	59	383	19	487	373	23	113
71	421	7	433	439	61	449	41	463	31
523	73	521	5	491	101	103	11	109	479

Fig. 12.

series (541) must be changed to 569 thus increasing the total to 24160. A magic square showing the lowest possible summation of 2416 was made with the above series by Mr. J. N. Muncey as reproduced in Fig. 12.

SQUARE OF THE ELEVENTH ORDER.

The sum of the first 121 primes, 1 to 661 inclusive, is 36887, which is not evenly divisible by 11. By substituting 677 for 659, the total is raised to 36905, the seventh part of which is 3355. This

1	613	3	587	61	631	107	643	19	653	37
607	73	71	223	619	59	569	47	503	43	541
461	467	83	347	137	499	53	509	149	127	523
457	181	593	173	179	167	491	157	409	151	397
211	353	197	487	227	431	239	419	271	269	251
331	199	337	307	283	293	421	277	389	379	139
317	463	443	67	349	229	373	233	257	241	383
193	313	311	439	433	359	163	367	113	401	263
97	89	449	617	479	103	281	31	557	521	131
79	599	191	101	11	571	17	563	647	547	29
601	5	677	7	577	13	641	109	41	23	661

Fig. 13.

series was arranged in magic formation by Mr. J. N. Muncey, and his square showing the lowest possible summation (3355) is given in Fig. 13.

SQUARE OF THE TWELFTH ORDER.

The sum of the first 144 prime numbers 1 to 827 inclusive, is 54168, the twelfth part of which is 4514. It is worthy of note that this is the only *straight series* of prime numbers of all that have been hitherto considered which is evenly divisible by *n*, and which is also capable of magic arrangement. Mr. J. N. Muncey has made a magic square of this series, which is shown in Fig. 14.

1	823	821	809	811	797	19	29	313	31	23	37
89	83	211	79	641	631	619	709	617	53	43	739
97	227	103	107	193	557	719	727	607	139	757	281
223	653	499	197	109	113	563	479	173	761	587	157
367	379	521	383	241	467	257	263	269	167	601	599
349	359	353	647	389	331	317	311	409	307	293	449
503	523	233	337	547	397	421	17	401	271	431	433
229	491	373	487	461	251	443	463	137	439	457	283
509	199	73	541	347	191	181	569	577	571	163	593
661	101	643	239	691	701	127	131	179	613	277	151
659	673	677	683	71	67	61	47	59	743	733	41
827	3	7	5	13	11	787	769	773	419	149	751

Fig. 14.

SUMMARY.

ORDER OF SQUARE	TOTALS OF SERIES	LOWEST SUMMATIONS	SQUARES MADE BY
3d	333	111	Henry E. Dudeney (1900)
4th	408	102	Ernest Bergholt and C. D. Shuldham
5th	1065	213	H. A. Sayles
6th	2448	408	C. D. Shuldham, J. N. Muncey
7th	4893	699	" "
8th	8912	1114	" "
9th	15129	1681	" "
10th	24160	2416	J. N. Muncey
11th	36905	3355	"
12th	54168	4514	"

W. S. ANDREWS.
H. A. SAYLES.

SCHENECTADY, N. Y.